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# A DATA STRUCTURE FOR MANIPULATING PRIORITY QUEUES

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Abstract :

We describe a data structure which can be used for representing a collection of priority queues. The primitive operations are insertion, deletion, union, update and search for an item of earliest priority.

Keywords :

Data structures - Implementation of set operations - Priority queues - Mergeable heaps - Binary trees.

CR categories : 4.0, 4.34, 5.24, 5.25, 5.32, 8.1.

## 1. INTRODUCTION.

In order to design correct and efficient algorithms for solving a specific problem, it is often helpful to describe our first approaches to a solution in a language close to that in which the problem was formulated. One such language is that of set theory, augmented by primitive set manipulation operations. Once the algorithm is outlined in terms of these set operations, one can then look for data structures best fitted for representing each of the sets involved. This choice only depends upon the collection of primitive operations required for each set. It is thus important to establish a good catalogue of such data structures, and a summary of the state of the art on this question can be found in AHU [2]. In this paper, we add to this catalogue a data structure which allows efficient manipulation of priority queues.

A priority queue is a set ; each element of such a set has a name and a label. Names are used to uniquely identify set elements. Labels, or priorities are drawn from a totally ordered set. Elements of the priority queue can be thought of as awaiting service, the item with the smallest label always being served next. Ordinary stacks and queues are special cases of priority queues.

A variety of applications directly require using priority queues : job scheduling and page replacement in operating systems, discrete simulation languages where labels represent the time at which events are to occur, as well as various sorting problems. These are discussed for example in AHU [2], C [4], G [11], JD [15], K [17], MS [19], VD [24] and W [27]. Priority queues also play a central role in several good algorithms such as Huffman's and Hu's optimal code constructions (see K [17] and C [4]), Chartres's prime number generator and Brown's power series multiplication (described in K [17]) ;

applications have also been found in numerical analysis algorithms (see G [10] and MS [19] for example) and in graph algorithms, for finding shortest paths (see D [5] , AHU [2] and J [13] ) and minimum cost spanning tree (see AHU [2] , CTY [3] , P [22] and V [25] ) among others.

Typical applications require primitive operations among the following five : INSERT, DELETE, MIN, UPDATE and UNION. The operation INSERT (name, label, Q) adds an element to queue Q while DELETE (name) removes the element having that name. Operation MIN (Q) returns the name of the element in Q having the least label, and UPDATE (name, label) changes the label of the element named. Finally, UNION ($Q_1$, $Q_2$) merges into $Q_3$ all elements of $Q_1$ and $Q_2$ ; the sets $Q_1$ and $Q_2$ become empty. In what follows, we assume that names are handled in a separate dictionnary (see AHU [2] and K [17] ) such as a hash-table or a balanced tree. If deletions are restricted to elements extracted by MIN, such an auxiliary symbol table can be dispensed with.

A truly elegant data structure, called a heap (see F [7] , GR [12] , K [17] , PS [21] ) has been discovered by J.W. Williams and R.W. Floyd ; it handles a sequence of n primitives INSERT, DELETE and MIN in 0 (n log n) elementary operations* and absolutely minimal storage. For applications in which UNION is necessary, more sophisticated data structures have been devised, such as 3-2 trees (see AHU [2] and K [17]), AVL trees (AL [1] and K [17] ), leftist trees (C [4] ), p-trees (JD [15] ) and binary heap (F [9] ).

The data structure we present here handles an arbitrary sequence of n primitives, each drawn among the five described above, in 0(n log n) machine operations and 0(n) memory cells. It also allows for an efficient treatment of a large number of updates, which is crucial in connection with spanning tree algorithms :

* We assume here that indexing through the symbol table is done in constant time.

our data structure provides an implementation (described in V [25 ]) of the Cheriton - Tarjan - Yao (CTY [3] ) minimum cost spanning tree algorithm which is much more straightforward than the original one.

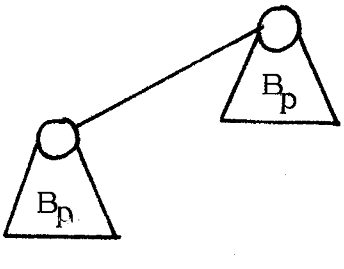
The proposed data structure uses less storage than leftist, AVL or 3-2 trees ; in addition, when the primitive operations are carefully machine coded from the programs given in paragraph 4, they yield worst case running times which compare favorably to those of their competitors.

Besides these technical advantages, we feel the data structure to be interesting in itself, because of its conceptual simplicity and of the connections it establishes between various data manipulation problems.

## 2. BINOMIAL TREES AND FORESTS.

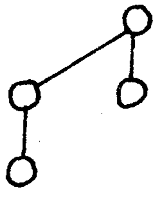
We describe here the underlying combinatorial structure, called binomial trees. These are defined inductively by :

$B_o$ = ○ and $B_{p+1}$ =

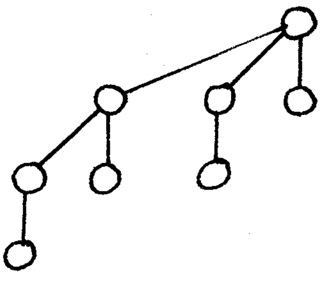


for p ≥ 0 .

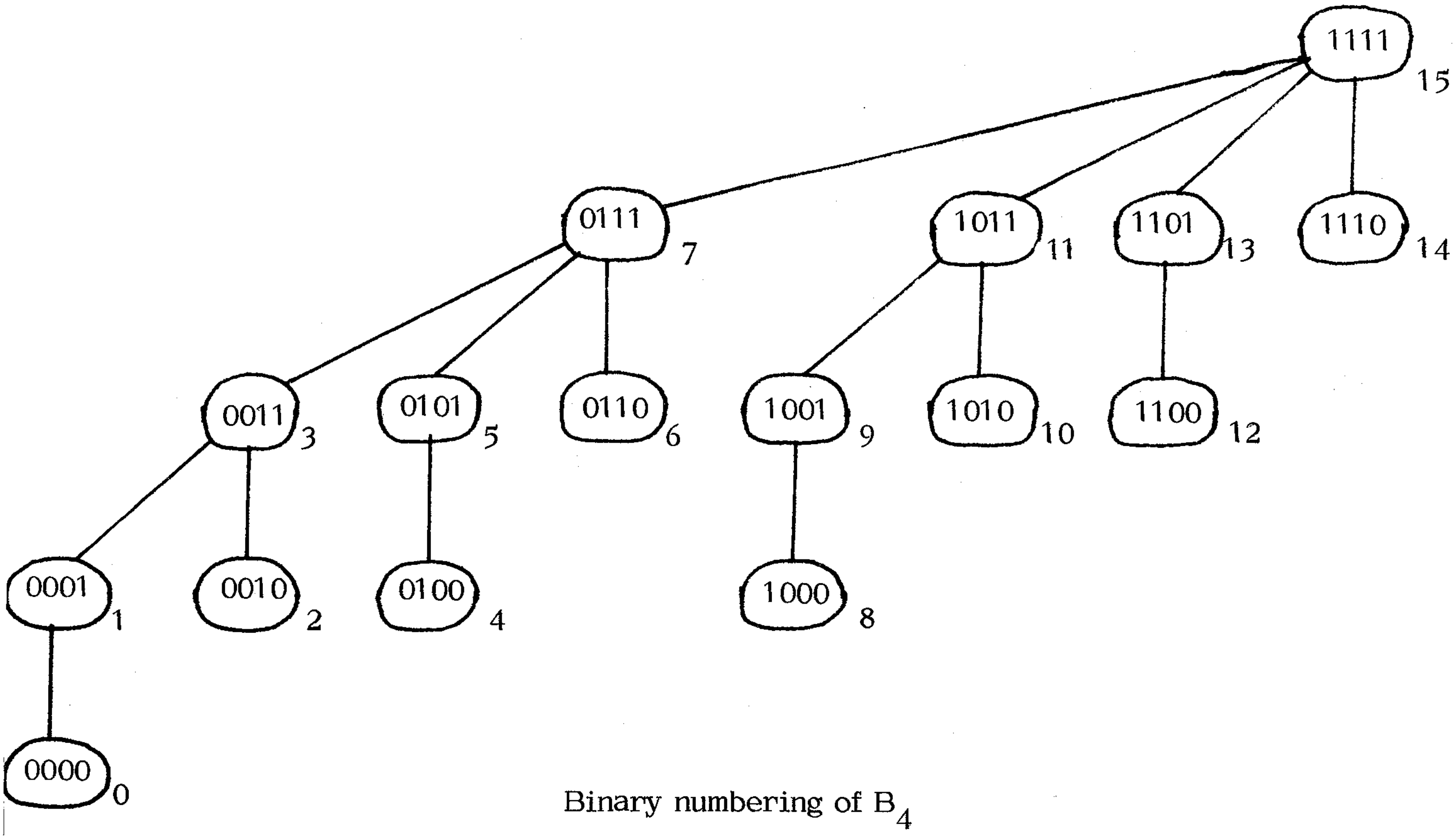
Thus $B_2$ =



, and $B_3$ =



for example. In order to discover some of the many combinatorial properties of the $B_i$ 's, Knuth (K [18 ]) suggests that we first label the nodes of the tree in post-order, starting at zero, then associate with each node the binary representation of its label.



Binary numbering of $B_4$

Figure 1.

From this numbering, it is easy to establish that

- each $B_p$ has $2^p$ nodes ;
- there are $\binom{p}{k}$ nodes at depth k in $B_p$ which correspond to the various sequences of p bits having exactly k zeros ;
- the maximum depth of a node in $B_p$ is p ;
- the number of children of a node is equal to the number of 1's following the last 0 in its binary numbering ; leaves thus correspond to even numbers ;
- in $B_p$, there is exactly one node, the root, having p children ; for $0 \leqslant k < p$ there are $2^{p-k-1}$ nodes having k children.

There are manv ways of drawing $B_p$ ; in particular :

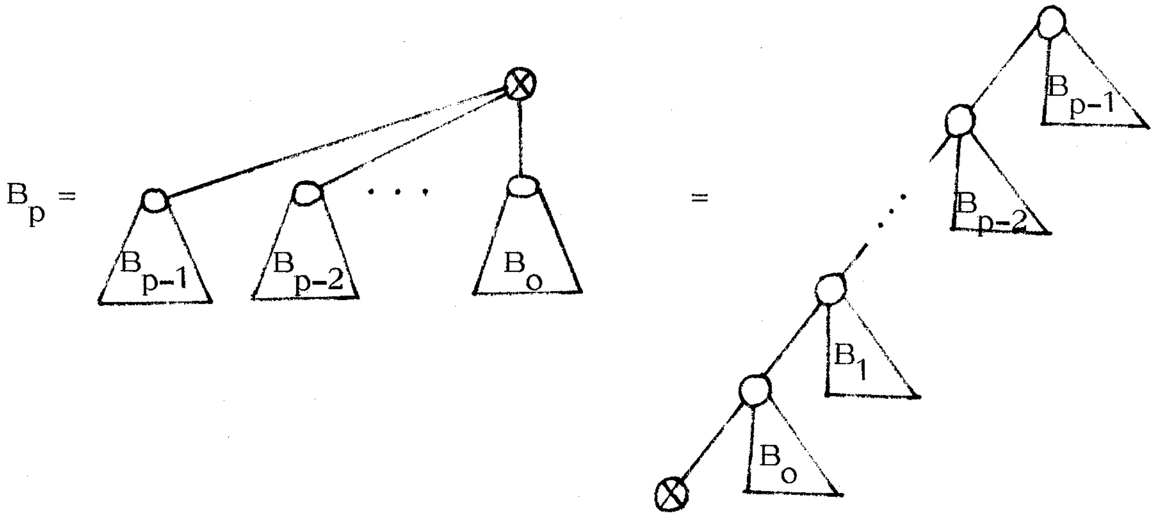


Figure 2

In order to use binomial trees for representing sets whose number n of elements is not always a power of two, we consider the binary decomposition $n = \sum_{i \geqslant 0} b_i 2^i$ with $b_i \in \{0, 1\}$ of the number n and define *a binomial forest $F_n$ of order n* as a finite collection of binomial trees, one $B_i$ for each 1 in the binary decomposition of n. In symbols, $F_n = \{ B_i \mid i \geqslant 0, b_i = 1, n = \sum_{i \geqslant 0} b_i 2^i \}$ ; we define the *i-th component of $F_n$* to be $B_i$ if $b_i = 1$ and empty otherwise. For example $(12)_{10} = (1100)_2$ thus $F_{12} = \{B_3, B_2\}$ ; the first component of $F_{12}$ is $\phi$ and its third $B_3$.

Figure 2 bis shows some small binomial forests.

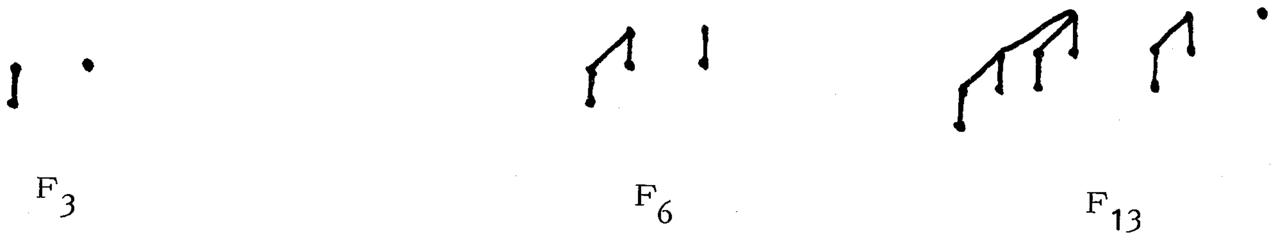
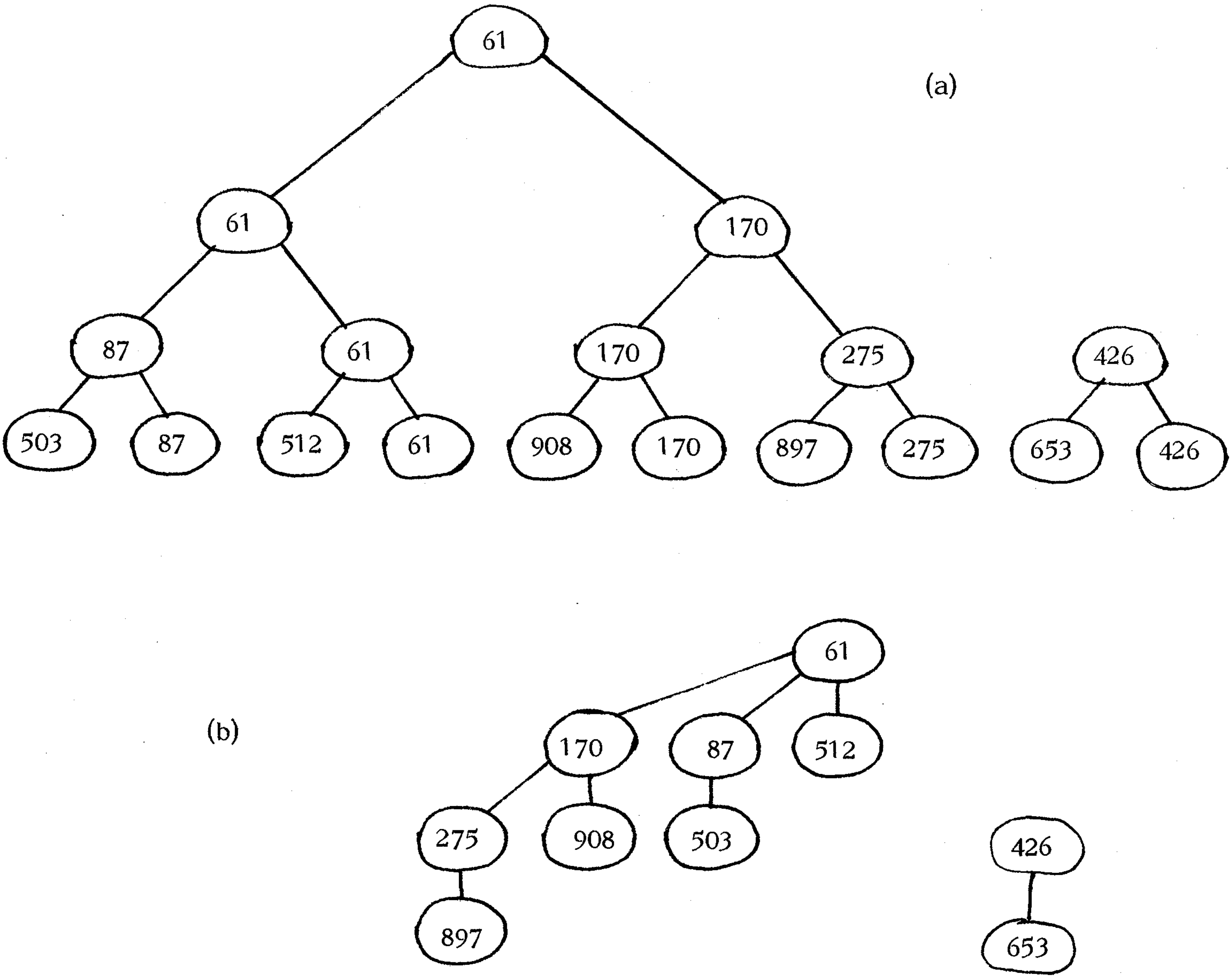


Figure 2 bis.

Binomial trees and forests appear in various data manipulation problems. They are used by Fisher (F [ 6 ) in the worst case analysis of a simple data structure for manipulating disjoint set unions (see also K [ 18 ] ). They play a crucial role in the linear time median algorithm of Paterson-Pippinger-Schönhage (PPS [ 20 ] ). The Ford-Johnson (FJ [8 ]) sorting algorithm can also be nicely described (non-recursively) with the help of binomial trees : the algorithm first builds a binomial forest, then sorts the partial order thus obtained through a sequence of repeated "foldings" ; the data structure presented here can actually by used for implementing the sorting algorithm in time 0(n log n). An efficient machine coding of binary search (see V [25] ) uses binomial search trees.

## 3. BINOMIAL QUEUES.

A priority queue $Q = \{<\nu_1,\lambda_1>, \ldots, <\nu_p,\lambda_p>\}$ consisting of p items is represented by a labeled binomial forest $F_p$ : each item < name, label > is stored in a different node of $F_p$ subject to the constraint that, if node i is a child of node j in $F_p$, the labels $\lambda_i$ and $\lambda_j$ of the items respectively associated must satisfy $\lambda_i \geqslant \lambda_j$. This is called the "heap condition" by Knuth (K [17] ), and it arises through the natural "contraction" of perfect tournaments as shown in figure 3 a) and b).



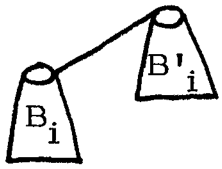
The collection $\{503, 87, 512, 61, 908, 170, 897, 275, 653, 426\}$ of labels represented as :

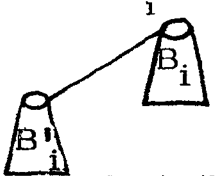
(a) a "perfect" tournament

(b) the same tournament "contracted" into a binomial queue $F_{10}$

Figure 3.

Such a labeled binomial forest will be called a binomial queue.

We now describe how to perform the UNION of two binomial queues $F_n$ and $F_{n'}$ . First consider the special case $n = n' = 2^i$, that is each priority queue is represented by a single labeled binomial tree, say $B_i$ for $F_n$ and $B'_i$ for $F_{n'}$. The resulting forest $B_{i+1}$ = UNION ($B_i$, $B'_i$) also consists of a single labeled binomial tree with $2^{i+1} = n + n'$ nodes, defined as $B_{i+1}$ = 

if the label of the root of $B'_i$ is smaller than the corresponding label in $B_i$ and $B_{i+1}$ = 

otherwise, in order to preserve the heap condition. We refer to this operation as coupling, and the general UNION procedure is a sequence of coupling.

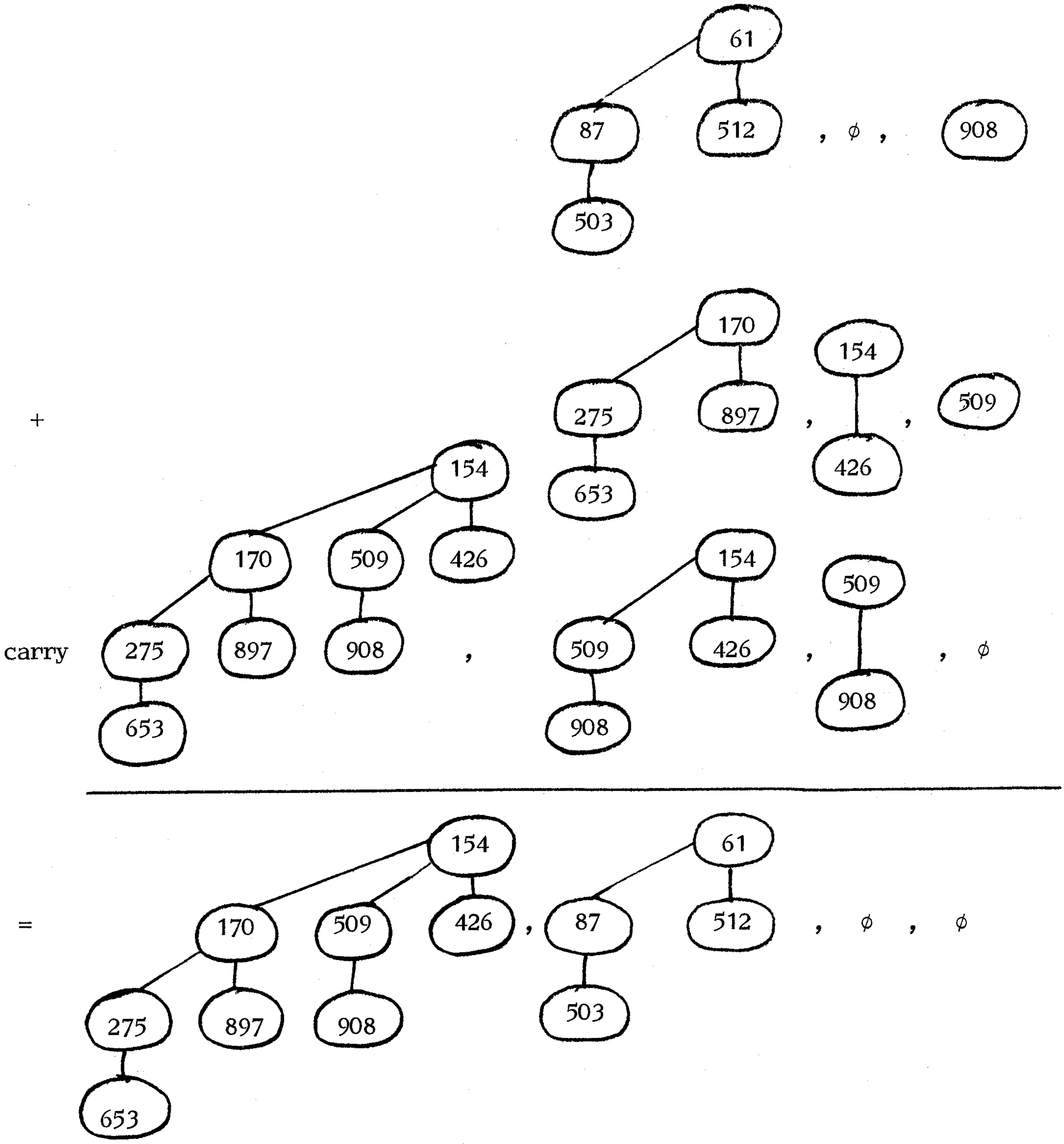
For treating the general case, i.e. for n and n' arbitrary, it is convenient to use an analogy with the ordinary scheme for the binary addition of n and n'. The UNION proceeds from low order bits to high order bits, i.e. it treats the binomial trees composing $F_n$ and $F_{n'}$ in order of increasing size. A carry is propagated at each step ; at the i-th step of the algorithm, the carry is either empty or it is a labeled binomial tree $B_i$. The initial carry is empty. Step i has three operands which play a symmetric role. Each operand is either empty or it is a labeled binomial $B_i$. One of the operands is the carry, and the other two are the i-th components of $F_n$ and $F_{n'}$ respectively. If all three operands are empty, the i-th component of the result UNION $(F_n, F_{n'})$ is empty as well as the carry propagated to the next step. If exactly one operand is non-empty, it constitutes the i-th component of the result and the carry is empty. If two operands are non empty , they are coupled according to the procedure described earlier in order to constitute the (i+1)-th carry ; the i-th component of the result is empty. When all three operands are non-empty, one of them arbitrarely constitutes the i-th component of the result and the remaining two are coupled in order to form the carry. The procedure starts at the 0-th step and stops when $i \geqslant 1 + \underline{\min} ( \lfloor \log_2 n \rfloor , \lfloor \log_2 n' \rfloor )$ and no carry is propagated any further. The algorithm is pictured below with $n = 7$ and $n' = 5$.

$$\begin{array}{lr} & 101 \\ + & 111 \\ \text{carry} & 111 \\ \hline = & 1100 \end{array} \qquad\qquad \begin{array}{lr} & 5 \\ + & 7 \\ \hline = & 12 \end{array}$$

(a)

$$\begin{array}{lcccc} & & B_2 & \emptyset & B_0 \\ + & & B_2 & B_1 & B_0 \\ \text{carry} & B_3 & B_2 & B_1 & \emptyset \\ \hline = & B_3 & B_2 & \emptyset & \emptyset \end{array}$$

(b)



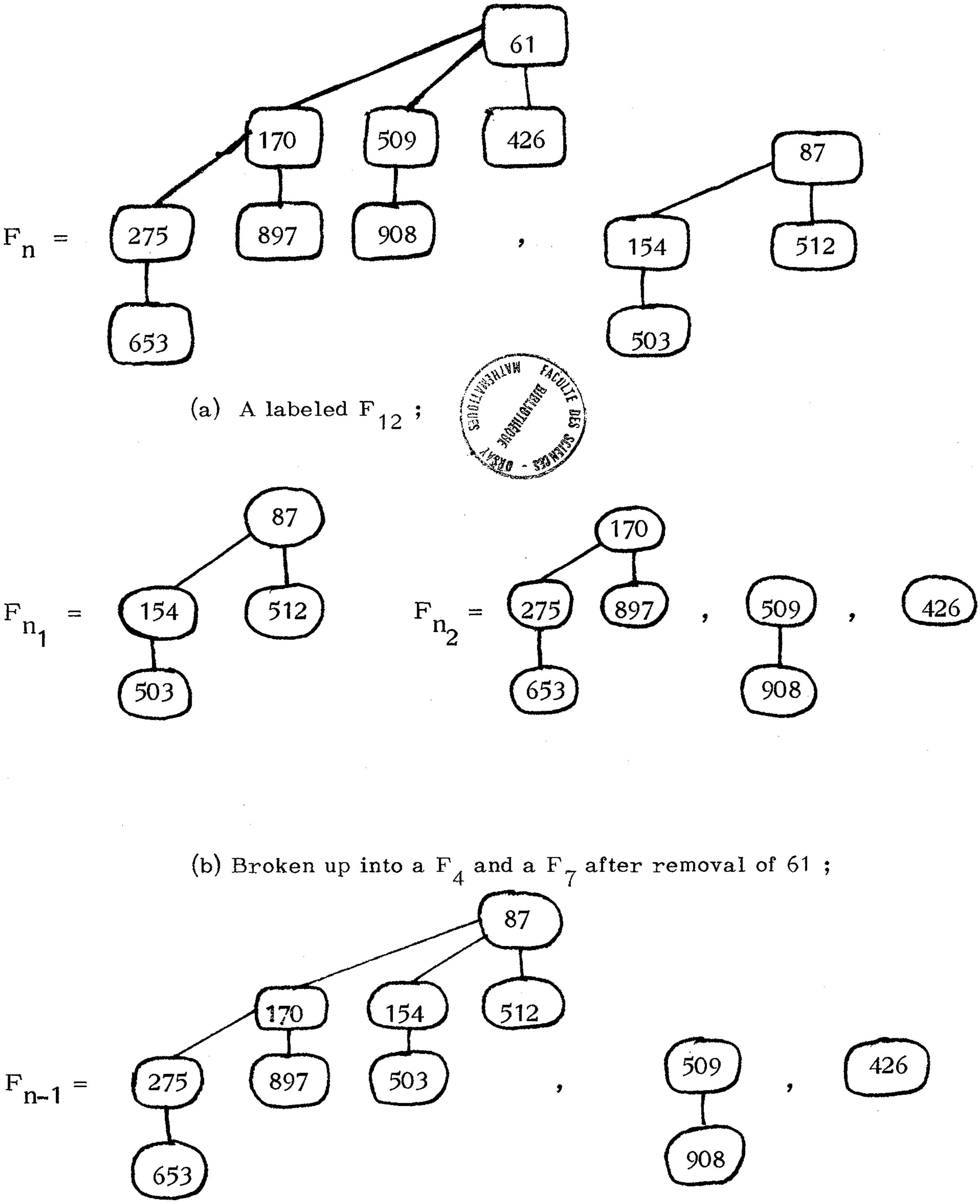
Binary addition (a) of 7 and 5, scheme (b) for UNION ($F_7$, $F_5$) and actual example (c)

Figure 4

By considering a single item as forming a $F_1$ binomial queue, INSERT can be treated as a special case of UNION. A forest $F_n$ is naturally constructed as the result of a sequence of n INSERT operations. The number of comparisons required by this construction is equal, on one hand to the number of carries propagated in the addition $\underbrace{1 + 1 + \ldots + 1}_{\text{n times}}$, and, on the other hand, to the number of edges in the graph of $F_n$. If $\nu$ (n) denotes the number of ones in the binary decomposition of n, this last number is clearly equal to $n - \nu(n)$. It follows that $F_n$ is constructed in $n-\nu(n)$ comparisons which is of linear order $O(n)$. As for UNION (n, n') exactly $\nu(n)+\nu(n')-\nu(n+n')$ comparisons are required, which is of order $O(\log (n + n'))$.

In order to find the minimal label of $F_n$, we merely have to explore the roots of the binomial trees composing $F_n$, and keep the name of a node having minimal label among these. This involves $\nu(n) - 1$ comparisons, which is of order $O(\log n)$.

In many applications, DELETE is restricted to extracting the item m found by MIN. Let $B_i$ be the labeled binomial tree in $F_n$ of which m is the root. We first remove $B_i$ from $F_n$, thus forming a labeled binomial forest $F_{n_1}$ with $n_1 = n - 2^i$. Then the root of $B_i$ is cut. As we can see from Figure 2, what remains is a "complete" forest $F_{n_2}$ with $n_2 = 2^i - 1$. One then call UNION $(F_{n_1}, F_{n_2})$ in order to restore $F_{n-1}$ in $O(\log n)$ comparisons again. This procedure is described on the example below (Figure 5).



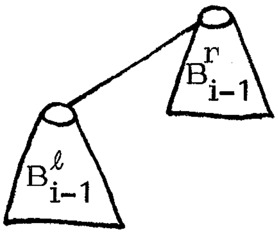
(a) A labeled $F_{12}$ ;

(b) Broken up into a $F_4$ and a $F_7$ after removal of 61 ;

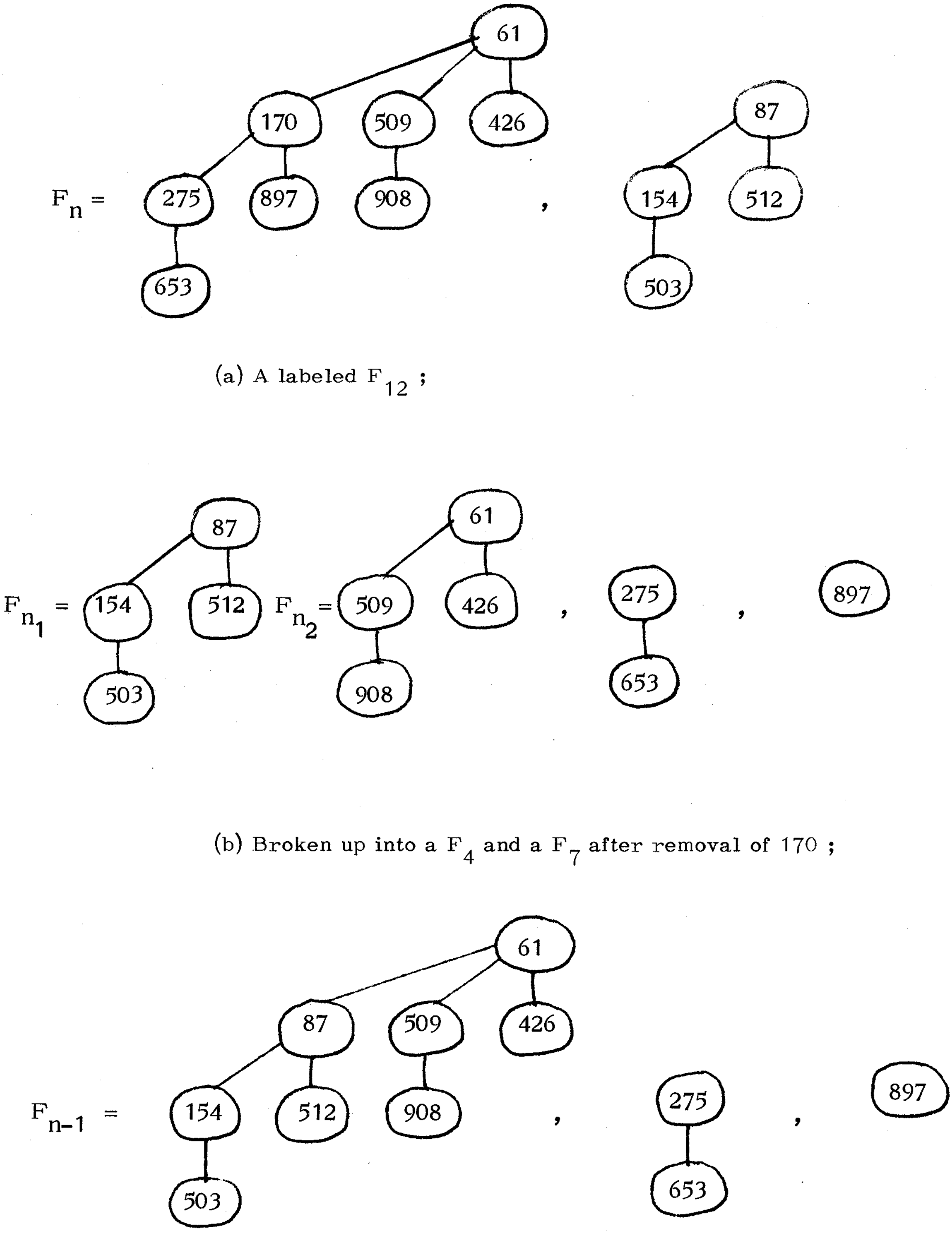
(c) $F_{11}$ = UNION ($F_4$, $F_7$) reconstituted.

Figure 5

If the item m to be removed by DELETE is not the root of one of the components of $F_n$, the algorithm is slightly more complex. First, we determine the component of $F_n$ in which m lies, say $B_i$. As before, we remove $B_i$ thus forming $F_{n_1}$ with $n_1 = n - 2^i$. We then consider $B_i$ =



If m is in $B^\ell_{i-1}$, we start constructing $F_{n_2} = \{B^r_{i-1}\}$ and decompose $B_{i-1}$ by the same technique ; otherwise, we further decompose $B^r_{i-1}$ and set $F_{n_2}$ to $B^\ell_{i-1}$. This continues until m becomes the root of the subtree $B_j$ to be decomposed. It is then "cut" from $B_j$, thus leaving a complete $F_{2^j-1}$ which is added to the binomial queue $F_{n_2} = \{B_{i-1}, B_{i-2}, \ldots, B_j\}$ already constructed in order to form a complete $F_{2^i-1}$. This forest is then merged with $F_{n_1}$, using the UNION procedure, in order to construct the resulting $F_{n-1}$. Again, this algorithm is illustrated by an example in Figure 6.



(a) A labeled $F_{12}$ ;

(b) Broken up into a $F_4$ and a $F_7$ after removal of 170 ;

(c) $F_{11}$ = UNION ($F_4$, $F_7$) reconstituted

Figure 6

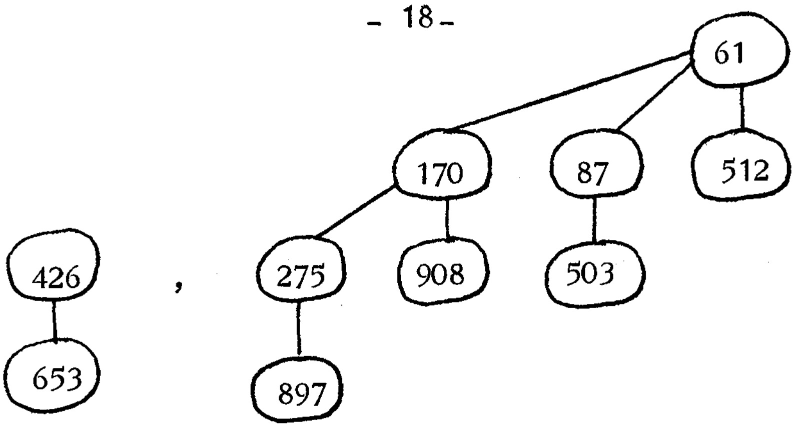
As for our last primitive, UPDATE, the obvious way to realize it is to perform DELETE then INSERT in sequence. This requires 0(log n) operations for each UPDATE . If we have to service an arbitrary sequence of primitive operations in which MIN is required less often than UPDATE and DELETE, there is a better way to proceed. The procedure UPDATE does not attempt to restore the structure, but merely changes the label and marks the node. The DELETE procedure proceeds in the same way, changing the label to, say $+\infty$ . The algorithm for MIN then becomes more complicated : it explores all binomial trees in the forest $F_n$ until finding unmarked nodes on all paths from the root of the tree to the leaves ; this cuts some subtrees and the forest is then reconstructed by merging all these subtrees together with the marked nodes having labels different from $+\infty$ . If $\mu$ marked nodes are met, it follows (see V [25] ) from the properties described in section 2 that the number of trees cut is a most $\mu \log \frac{n}{\mu}$ . If we consider an arbitrary sequence of n INSERT or UNION, u UPDATE or DELETE, and m MIN, an elementary analysis (see V [25] again) shows that such a sequence is treated in at most $0(n \log n) + 0(u) + 0(u \log \frac{n\,m}{u})$ operations which is better than the naïve method for $m \leq u$.

Yet another way of treating UPDATE is described in J [14] .

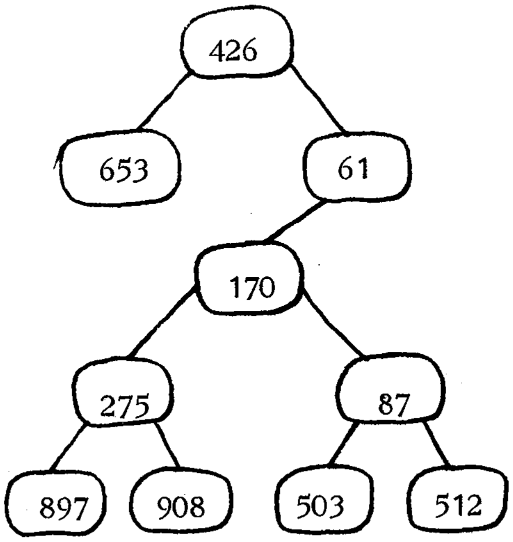
## 4. IMPLEMENTATION OF BINOMIAL FORESTS AS BINARY TREES.

While the above discussion constitutes an adequate presentation of the priority queue primitives if our machine model is rather abstract (decision trees for example) it is by no means complete if one is to actually code these algorithms on a real life computer. One still has to solve some problems, the first of which concerns the machine representation of labeled binomial forests.

For this purpose, we represent binomial forests as binary trees through the well known "natural correspondence" described in K [16] : each node has fields *llink* and *rlink* such that llink points to the leftmost child of the node and rlink to the node's right sibling. This leaves some freedom for defining a node's right sibling. For the purpose of the UNION procedure, it is crucial to link small trees to larger siblings on the top level, i.e. for nodes having no parent, and to link large trees to smaller siblings on lower levels, as shown in Figure 7.



(a) A binomial queue $F_{10}$ ;



(b) The same $F_{10}$ as a binary tree ;

| R |
|---|
| 4 |

| N |
|---|
| 10 |

|  | INFO | LLINK | RLINK |
|---|---|---|---|
| 1 | 512 | 0 | 0 |
| 2 | 87 | 9 | 1 |
| 3 | 61 | 10 | 0 |
| 4 | 426 | 6 | 3 |
| 5 | 275 | 7 | 8 |
| 6 | 653 | 0 | 0 |
| 7 | 897 | 0 | 0 |
| 8 | 908 | 0 | 0 |
| 9 | 503 | 0 | 0 |
| 10 | 170 | 5 | 2 |

(c) A possible machine representation for this binary tree, using arrays ; memory cells R and N contain respectively a pointer to the root of the tree and its number of elements.

Figure 7 .

We can now give a formal description of our algorithms in an Algol-like language. Following Knuth (K [16] ), we represent binary tree with three arrays INFO, LLINK, RLINK containing respectively the label, llink and rlink of the node under consideration. The value 0 represents an empty pointer.

We first describe the UNION procedure :

```
proc UNION(R1,N1,R2,N2)→(R3,N3):
    {This procedure merges the two binomial queues $F^1$ and $F^2$, yielding $F^3$ for
      result. Each $F^i$ is represented as a binary tree : $R_i$ is a pointer to its
      root and $N_i$ represents the number of elements in $F^i$.
      The initial carry C is zero. Variable RES points to the part of $F^3$ being
      currently constructed. Location RLINK[0] is used to keep $R_3$ and thus
      should be available upon calling the procedure.}

    (N3,C,RES)←(N1+N2,0,0);
    while (min(N1,N2)≠0)∨(C≠0)
    do NEXTBIT;(N1,N2)←(⌊N1/2⌋,⌊N2/2⌋) od;
    RLINK[RES]←if N1≠0 then R1 else R2 fi;
    R3←RLINK[0]
endproc UNION.
```

Here NEXTBIT stands for a fragment of program that treats the eight possible cases for the carry and the relevant component of $F^1$ and $F^2$ :

```
macro NEXTBIT :
        BITC←if C=0 then 0 else 1 fi;
        case < N1 mod 2,N2 mod 2,BITC>
              000:
              001: (C,RES,RLINK[RES])←(0,C,C)
              010: PROPRES(R2)
              011: PROPCARRY(R2)
              100: PROPRES(R1)
              101: PROPCARRY(R1)
              110: CONSTRUCTCARRY
              111: PROPRES(R1);PROPCARRY(R2)
        endcase
endmacro NEXTBIT.
```

We then have to describe the various macro procedures composing NEXTBIT :

```
macro PROPRES(R):
{The number of bits is odd and one of then, namely R, must be added to the result.}
        (R,RES,RLINK[RES])←(RLINK[R],R,R)
endmacro PROPRES.
```

```
macro PROPCARRY(R):
{Bit R must be added to the carry and the carry propagated.}
if INFO[R]< INFO[C]
  then (C,R,LLINK[R],RLINK[C])←(R,RLINK[R],C,LLINK[R])
  else (R,LLINK[C],RLINK[R])←(RLINK[R],R,LLINK[C])
fi
endmacro PROPCARRY.
```

```
macro CONSTRUCTCARRY :
{Bits R1 and R2 are on and a carry must be constructed.}
if INFO[R1]<INFO[R2]
  then (C,R1,R2,LLINK[R1],RLINK[R2])←(R1,RLINK[R1],RLINK[R2],R2,LLINK[R1])
  else (C,R2,R1,LLINK[R2],RLINK[R1])←(R2,RLINK[R2],RLINK[R1],R1,LLINK[R2])
fi
endmacro CONSTRUCTCARRY.
```

This terminates the description of UNION. The procedure MIN being straightforward, we omit its description. (If very frequent uses of MIN are requested, we can keep the value of the minimal label in a special register.)

As for DELETE, we simply treat EXTRACTMIN where the element having least label is first found, then removed. (The DELETE procedure for which we give no formal code, is very similar. A little complication arises from the necessity of keeping and updating upward parent links.)

```
proc EXTRACTMIN(R,N)→(R',N'):
     {This procedure extracts from the non-empty labeled binomial forest F, the
       element having minimal label.  The resulting forest F' is obtained by merging
       two forests F^1 and F^2 which we first construct.}
     (M,PRED,R2,P,R)←(R,0,R,R,RLINK[R]);
     while R≠0
     do    if INFO[R]<INFO[M] then (M,PRED)←(R,P) fi;
           (R,P)←(RLINK[R],R)
     od    {INFO[M] is the minimal label in F.}
     (R1,N1)←CONSTRUCT(LLINK[M]); N2←N-N1-1;
     if PRED=0 then R2←RLINK[M] else RLINK[PRED]←RLINK[M] fi;
     (R',N')←UNION(R1,N1,R2,N2)
endproc EXTRACTMIN.
```

The procedure EXTRACTMIN uses CONSTRUCT which transforme the binary tree representations of a $B_p$ into the binary tree representation of the complete forest $F_{2^p-1}$ obtained by removing the root of $B_p$.

```
macro CONSTRUCT(RAC)→(R,N):
if RAC=0
  then (R,N)←(0,0)
  else (R,SUCC,RLINK[RAC],P)←(RAC,RLINK[RAC],0,1);
       while SUCC≠0
       do (R,SUCC,RLINK[SUCC],P)←(SUCC,RLINK[SUCC],R,2×P) od;
       N←2×P-1
fi
endmacro CONSTRUCT.
```

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